

## 0.1 poisson: Poisson Regression for Event Count Dependent Variables

Use the Poisson regression model if the observations of your dependent variable represents the number of independent events that occur during a fixed period of time (see the negative binomial model, Section ??, for over-dispersed event counts.) For a Bayesian implementation of this model, see Section ??.

### Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "poisson", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

### Additional Inputs

In addition to the standard inputs, `zelig()` takes the following additional options for poisson regression:

- **robust**: defaults to `FALSE`. If `TRUE` is selected, `zelig()` computes robust standard errors via the `sandwich` package (see Zeileis (2004)). The default type of robust standard error is heteroskedastic and autocorrelation consistent (HAC), and assumes that observations are ordered by time index.

In addition, **robust** may be a list with the following options:

- **method**: Choose from
  - \* **"vcovHAC"**: (default if **robust** = `TRUE`) HAC standard errors.
  - \* **"kernHAC"**: HAC standard errors using the weights given in Andrews (1991).
  - \* **"weave"**: HAC standard errors using the weights given in Lumley and Heagerty (1999).
- **order.by**: defaults to `NULL` (the observations are chronologically ordered as in the original data). Optionally, you may specify a vector of weights (either as **order.by** = `z`, where `z` exists outside the data frame; or as **order.by** = `~z`, where `z` is a variable in the data frame). The observations are chronologically ordered by the size of `z`.
- **...**: additional options passed to the functions specified in **method**. See the `sandwich` library and Zeileis (2004) for more options.

### Example

Load sample data:

```
> data(sanction)
```

Estimate Poisson model:

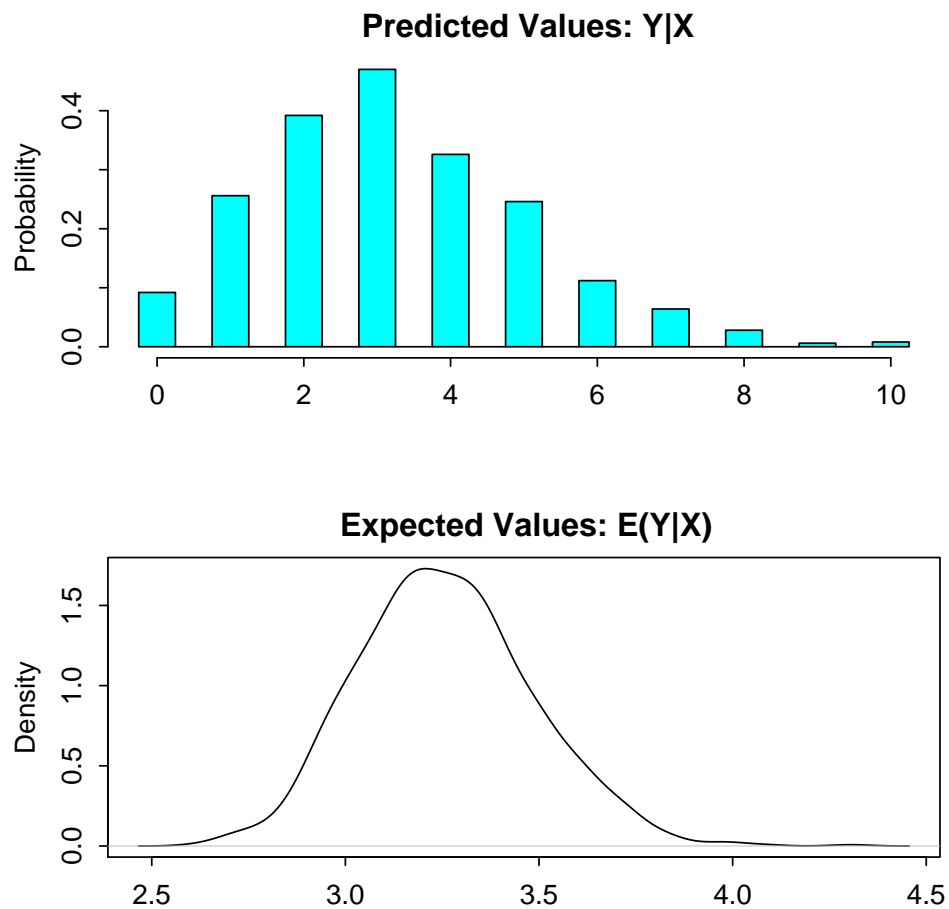
```
> z.out <- zelig(num ~ target + coop, model = "poisson", data = sanction)
> summary(z.out)
```

Set values for the explanatory variables to their default mean values:

```
> x.out <- setx(z.out)
```

Simulate fitted values:

```
> s.out <- sim(z.out, x = x.out)
> summary(s.out)
> plot(s.out)
```



## Model

Let  $Y_i$  be the number of independent events that occur during a fixed time period. This variable can take any non-negative integer.

- The Poisson distribution has *stochastic component*

$$Y_i \sim \text{Poisson}(\lambda_i),$$

where  $\lambda_i$  is the mean and variance parameter.

- The *systematic component* is

$$\lambda_i = \exp(x_i\beta),$$

where  $x_i$  is the vector of explanatory variables, and  $\beta$  is the vector of coefficients.

## Quantities of Interest

- The expected value (**qi\$ev**) is the mean of simulations from the stochastic component,

$$E(Y) = \lambda_i = \exp(x_i\beta),$$

given draws of  $\beta$  from its sampling distribution.

- The predicted value (**qi\$pr**) is a random draw from the poisson distribution defined by mean  $\lambda_i$ .
- The first difference in the expected values (**qi\$fd**) is given by:

$$\text{FD} = E(Y|x_1) - E(Y|x)$$

- In conditional prediction models, the average expected treatment effect (**att.ev**) for the treatment group is

$$\frac{1}{\sum_{i=1}^n t_i} \sum_{i:t_i=1}^n \{Y_i(t_i = 1) - E[Y_i(t_i = 0)]\},$$

where  $t_i$  is a binary explanatory variable defining the treatment ( $t_i = 1$ ) and control ( $t_i = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $E[Y_i(t_i = 0)]$ , the counterfactual expected value of  $Y_i$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $t_i = 0$ .

- In conditional prediction models, the average predicted treatment effect (**att.pr**) for the treatment group is

$$\frac{1}{\sum_{i=1}^n t_i} \sum_{i:t_i=1}^n \left\{ Y_i(t_i = 1) - \widehat{Y_i(t_i = 0)} \right\},$$

where  $t_i$  is a binary explanatory variable defining the treatment ( $t_i = 1$ ) and control ( $t_i = 0$ ) groups. Variation in the simulations are due to uncertainty in simulating  $\widehat{Y_i(t_i = 0)}$ , the counterfactual predicted value of  $Y_i$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $t_i = 0$ .

## Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "poisson", data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:
  - `coefficients`: parameter estimates for the explanatory variables.
  - `residuals`: the working residuals in the final iteration of the IWLS fit.
  - `fitted.values`: a vector of the fitted values for the systemic component  $\lambda$ .
  - `linear.predictors`: a vector of  $x_i\beta$ .
  - `aic`: Akaike's Information Criterion (minus twice the maximized log-likelihood plus twice the number of coefficients).
  - `df.residual`: the residual degrees of freedom.
  - `df.null`: the residual degrees of freedom for the null model.
  - `zelig.data`: the input data frame if `save.data = TRUE`.
- From `summary(z.out)`, you may extract:
  - `coefficients`: the parameter estimates with their associated standard errors,  $p$ -values, and  $t$ -statistics.
  - `cov.scaled`: a  $k \times k$  matrix of scaled covariances.
  - `cov.unscaled`: a  $k \times k$  matrix of unscaled covariances.
- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation  $\times$   $\mathbf{x}$ -observation (for more than one  $\mathbf{x}$ -observation). Available quantities are:
  - `qi$ev`: the simulated expected values given the specified values of  $\mathbf{x}$ .
  - `qi$pr`: the simulated predicted values drawn from the distributions defined by  $\lambda_i$ .
  - `qi$fd`: the simulated first differences in the expected values given the specified values of  $\mathbf{x}$  and  $\mathbf{x1}$ .

- `qi$att.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.
- `qi$att.pr`: the simulated average predicted treatment effect for the treated from conditional prediction models.

## How to Cite

To cite the *poisson* Zelig model:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “poisson: Poisson Regression for Event Count Dependent Variables,” in Kosuke Imai, Gary King, and Olivia Lau, “Zelig: Everyone’s Statistical Software,” <http://gking.harvard.edu/zelig>.

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Zelig: Everyone’s Statistical Software,” <http://GKing.harvard.edu/zelig>.

Kosuke Imai, Gary King, and Olivia Lau. 2007. “Toward A Common Framework for Statistical Analysis and Development,” <http://gking.harvard.edu/files/abs/z-abs.shtml>.

## See also

The *poisson* model is part of the *stats* package by Venables and Ripley (2002). Advanced users may wish to refer to `help(glm)` and `help(family)`, as well as McCullagh and Nelder (1989). Robust standard errors are implemented via the *sandwich* package by Zeileis (2004). Sample data are from Martin (1992).

# Bibliography

- Andrews, D. W. (1991), “Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation,” *Econometrica*, 59, 817–858.
- Lumley, T. and Heagerty, P. (1999), “Weighted Empirical Adaptive Variance Estimators for Correlated Data Regression,” *jrssb*, 61, 459–477.
- Martin, L. (1992), *Coercive Cooperation: Explaining Multilateral Economic Sanctions*, Princeton University Press, please inquire with Lisa Martin before publishing results from these data, as this dataset includes errors that have since been corrected.
- McCullagh, P. and Nelder, J. A. (1989), *Generalized Linear Models*, no. 37 in Monograph on Statistics and Applied Probability, Chapman & Hall, 2nd ed.
- Venables, W. N. and Ripley, B. D. (2002), *Modern Applied Statistics with S*, Springer-Verlag, 4th ed.
- Zeileis, A. (2004), “Econometric Computing with HC and HAC Covariance Matrix Estimators,” *Journal of Statistical Software*, 11, 1–17.